



Faculty of Engineering

# **FINITE ELEMENT ANALYSIS OF SEEPAGE FLOW UNDER A SHEET PILE**

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Bachelor of Engineering with Honours  
(Civil Engineering)

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**Universiti Malaysia Sarawak**  
Kota Samarahan

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This project is submitted in partial fulfillment of  
the requirements for the degree of Bachelor of Engineering with Honours  
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Dedicated To My Parent, Lecturers and Friends

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## **ABSTRACT**

A study on the phenomenon of seepage flow under a sheet pile using the finite element method is presented. It has been hypothesized that, the usage of sheet pile is an effective way of seepage controlling measure by which it will significantly reduce the rate of seepage flow. A finite element model has been developed to analyze the seepage flow under a sheet pile. The element used was a four-node quadrilateral element. The flow domain was discretised into suitable elements and the flow potential head at each node and velocity components at each element were obtained. The total discharges of seepage flow under a sheet pile were found. The present study tends to analyze the variation of results of seepage flow under a sheet pile for three different length sheet piles with two different cases of soil conditions. A FORTRAN program was developed to analyze the problem.

## **ABSTRAK**

Kajian ini adalah tentang phenomena penyisipan aliran air menerusi satu 'sheet pile' di bawah tanah dengan penggunaan kaedah unsur terhingga. Hipotesis menyatakan bahawa penggunaan 'sheet pile' merupakan satu cara yang efektif dalam pencegahan penyisipan aliran air in dalam tanah. Satu model unsur terhingga telah dibuat untuk kajian ini. Empat-nodal 'quadrilateral' elemen telah diwujudkan dan digunakan dalam penganalisaan penyisipan aliran air menerusi satu 'sheet pile' di bawah tanah. Region pengaliran air dalam kajian telah dibahagikan kepada elemen kecil yang sesuai, potensi pengairan air pada setiap nodal dan halaju air dalam setiap elemen telah didapatkan. Kadar penyisipan aliran air menerusi satu 'sheet pile' di bawah tanah telah dikirakan. Kajian ini juga cuba mendapatkan variasi jawapan bagi phenomena penyisipan aliran air menerusi satu 'sheet pile' di bawah tanah dengan penggunaan tiga panjang 'sheet pile' yang berbeza di bawah dua kes tanah yang berbeza. Satu program Fortran telah dihasilkan bagi menganalisa kajian ini.



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## LIST OF SYMBOLS

$Q$	Total discharge of seepage
$A$	Area
$K$	Element stiffness matrix or coefficient of permeability
$L$	Length
$i$	Hydraulic conductivity
$V$	Velocity
$\phi$	Potential head
$N$	shape function or interpolation function
$\eta, \xi$	Local coordinate system
$a, b$	Mesh dimension
$g_x, g_y$	Gradient potential
$[B]$	Gradient-potential transformation matrix
$k_x, k_y$	Horizontal and vertical permeability of soil material
$D$	Soil properties matrix
$NEL$	Number of element
$NNP$	Number of nodes point
$Nomat$	Number of soil material
$Mtype$	Soil material type

# CHAPTER 1

## INTRODUCTION

### 1.1 General

The flow of water through soil is one of the fundamental issues in geotechnical and geo-environmental engineering. Flow quantity is often considered to be the key parameter in quantifying seepage losses from a reservoir or determining the amount of water available for domestic or industrial use. In engineering, the more important issue is the pore-water pressure. The emphasis should not be on how much water is flowing through the ground, but on the state of the pore-water pressure in the ground. The pore-water pressure, whether positive or negative, has a direct bearing on the shear strength and volume change characteristics of the soil. Research in the last few decades has shown that even the flow of moisture in the unsaturated soil near the ground surface is directly related to the soil suction (negative water pressure). So, even when flow quantities are the main interest, it is important to accurately establish the pore-water pressures.



In the past, the analyses related to groundwater have concentrated on saturated flow. As a result, flow problems were typically categorized as being confined and unconfined situations, such as confined or unconfined aquifers. Flow beneath a structure would be a confined flow problem, while flow through a homogeneous embankment would be unconfined flow. Historically speaking, unconfined flow problems were more difficult to analyze because the analysis required determining the 'phreatic surface' or 'free surface'. The phreatic surface was considered an upper boundary and any flow that may have existed in the capillarity zone above the phreatic line was ignored.

It is no longer acceptable to take a simplified approach and ignore unsaturated flow above the phreatic surface. Not only does it ignore an important component of moisture flow in soils, but it greatly limits the types of problems that can be analyzed. It is mandatory to deal with unsaturated flow in typical situations such as modeling infiltration of precipitation. Transient flow problems are another good example. It is nearly impossible to model a situation where a wetting front moves through an earth structure without correctly considering the unsaturated component of flow. Fortunately, it is no longer necessary to ignore the unsaturated zone. With the help of the associated software, unsaturated flow can be considered in numerical modeling and the door is opened to analyzing almost any kind of seepage problem.

The term 'seepage' usually refers to situations where the primary driving force is

gravity controlled, such as establishing seepage losses from a reservoir, where the driving force is the total hydraulic head difference between the entrance and exit points. Another cause of water movement in soils is the existence of excess pore-water pressure due to external loading. This type of water flow is usually not referred to as seepage, but the fundamental mathematical equations describing the water movement are essentially identical. Thus, the term seepage can be used to describe all movement of water through soil regardless of the creation or source of the driving force or whether the flow is through saturated or unsaturated soils.

Modeling the flow of water through soil with a numerical solution can be very complex. Natural soil deposits are generally highly heterogeneous and nonisotropic. In addition, boundary conditions often change with time and cannot always be defined with certainty at the beginning of an analysis; in fact, the correct boundary condition can sometimes be part of the solution. Furthermore, when a soil becomes unsaturated, the coefficient of permeability or hydraulic conductivity becomes a function of the negative pore-water pressure in the soil. The pore-water pressure is the primary unknown and needs to be determined, so iterative numerical techniques are required to match the computed pore-water pressure and the material property, which makes the solution highly non-linear. These complexities make it necessary to use some form of numerical analysis to analyze seepage problems for all, but the simplest cases. A common approach is to use finite element formulations.

## **1.2 Project Objective**

The present study is primarily concerned with the study and analysis of seepage flow under a sheet pile by numerical analysis. In the present study, it is intend to use Finite Element Analysis Method. The advantage of this method is that once a particular problem is identified and necessary formulation is made, a FORTRAN program is developed for the problem can often treated as very general because by simply inserting the numerical values for the boundary conditions of a problem the solution may be effectively obtained by the use of digital computer. In the present study, it is intended to find out the following objectives:

- i. The potential head, flow velocity and total discharge of the flow under 3 different length of sheet pile.
- ii. The variations of result for potential head, flow velocity and total discharge under specified conditions
- iii. The effectiveness of sheet pile as a medium of controlling seepage flows.

## **1.3 Report structure**

This report is divided into 5 chapters. It provides an overview of seepage flow problem and addresses the method available for analysis of seepage flows, which followed by finite element analysis of seepage flow under a sheet pile.

Chapter 2 discusses the principles of flow through porous media which is governed by Darcy's laws and continuity equation. It also covers the overview of various seepage flow analysis methods that have been used.

Chapter 3 presents methodologies that have been used in analyzing the seepage flow under a sheet pile with specified conditions, which may cover finite element method, procedures in finite element analysis, finite element formulation and outline the three distinct steps connected with input, analysis and output through Pre-processing, Processing and Post-processing.

Chapter 4 discusses the outputs that obtained in FORTRAN program and make the necessary computation. Besides that, also discusses the comparisons of result for seepage flow under a sheet pile with 3 different sheet piles length.

Chapter 5 presents the conclusions from the present study and recommendations for further development.

# CHAPTER 2

## LITERATURE REVIEW

### 2.1 Historical Background

Seepage problems are among the most commonly analyzed problems in geotechnical engineering. Solution of water movement (seepage) according to Casagrande (1937), he presented a complete discussion on the use of the flow net technique for predicting seepage through earth structures, originally developed by Forchheimer. Cassagrande (1937) divided the soil into two parts, the soil below the water table and the soil above the water. The assumption was made that water only flowed below the water table. The flow net method was used extensively in geotechnical practice. Various investigators (Taylor and Brown, 1967; Freeze, 1971) developed finite element models for describing water flow and seepage in soils. Papagianakis and Fredlund (1984) and Lam et al. (1987) developed a finite element package for performing saturated/unsaturated seepage modeling. Nguyen (1999) showed that it is possible to use a general partial differential equation solver for modeling seepage in saturated/unsaturated soils. The finite element method has

essentially replaced the flow net method for solving seepage problem, due to the robust nature of numerical modeling software.

## **2.2 Principles of Flow through Porous Media**

### **2.2.1 Darcy's Law**

Darcy, a well known French hydraulic engineer, conducted numerous experiments in a vertical pipe with sand under steady state saturated seepage conditions and observed the  $Q$  was proportional to area of cross section of pipe  $A$  containing sand column and head causing flow,  $H$  and to the reciprocal of the column length of sand  $L$ . Thus,

$$Q = \frac{KAH}{L}$$

Hence, the discharge velocity through the porous medium is given by

$$V = \frac{Q}{A} = \frac{KH}{L} = K_1 \text{ or}$$

$$V = Ki$$

Where,

$\frac{H}{L}$  is the hydraulic gradient which represents the rate of energy dissipation

$K$  is the constant proportionality

This equation demonstrates a linear dependency between the hydraulic gradient

and the discharge velocity (V). The coefficient of proportionality is also known as coefficient permeability.

### 2.2.2 Laplace Equation

A homogenous isotropic medium is characterized by a constant value of hydraulic conductivity K in all directions at a points and the same value at all points in field. Hence,

$$V_x = -K \frac{\partial \phi}{\partial x}, V_y = -K \frac{\partial \phi}{\partial y}, V_z = -K \frac{\partial \phi}{\partial z} \quad (2.1)$$

Where  $V_x$ ,  $V_y$ ,  $V_z$  denote the components of velocity (V) in Cartesian coordinate direction x, y, and z directions respectively and  $\phi$  denotes the piezometric head at any point of the seepage region measured above any reference level. The equation of continuity for the steady flow incompressible fluids which expresses the law of conservation of mass gives

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad (2.2)$$

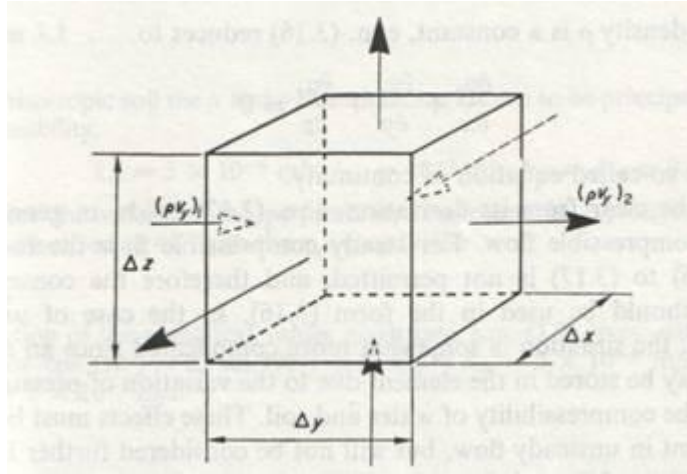


Figure 2.1 Specific mass discharges into and out of element (law of conservation of mass)

Substituting the values of  $V_x, V_y, V_z$  of equation (2.1) into the continuity equation (2.2)

one obtains Laplace's equation of the piezometric head

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2.3)$$

From the equation (2.1) it follows that the projections of seepage velocity may be expressed as derivatives of the function,  $\Phi = -K\phi$  called the velocity potential, i.e.,

$$V_x = \frac{\partial \Phi}{\partial x}, V_y = \frac{\partial \Phi}{\partial y}, V_z = \frac{\partial \Phi}{\partial z} \quad (2.4)$$

Substituting the velocity components given by equation (2.4) into continuity equation (2.2) one obtain the Laplace's equation for seepage velocity potential

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad (2.5)$$